*Journal of Statistical Physics, Vol. 88, Nos. 3/4, 1997*

# On a Generalized Model of Biological Evolution

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*Received November 18, 1996; final March 13, 1997*

A model for biological evolution with relative fitness between different species is proposed. It contains both negative interactions, e.g., predation, competition, etc., and positive interactions, e.g., mutualism, sharing, etc. This is called coevolution.

**KEY WORDS**: Biological evolution model; coevolution.

## 1. INTRODUCTION

Self-organized criticality (SOC)[1,2] offers a new and interesting approach to the study of dissipative, self-sustaining systems with many degrees of freedom. Therefore it has many application in physics, geology, economics, etc. Due to the complexity of such systems, very few studies have been done to elucidate the mathematical structure of SOC.[3] Presently the most effective approach is modeling. It is important to note that since SOC systems have many degrees of freedom, the concept complements chaos[4] rather than being included in it. Therefore it may be important to formulate SOC models for practical systems, e.g., meteorology, biology, etc.

Recently Bak and Sneppen (BS) used SOC to model biological evolution. The $N$ species of an ecosystem is each characterized by a positive number between zero and one called the fitness of the species; the higher the fitness, the better adapted the species is to the ecosystem. Also, the species with the lowest fitness in the system will be the next to mutate or go extinct. In the BS model, the $N$ species are arranged on a line. At each step, the species with the lowest fitness and its nearest neighbors mutate, i.e., their fitnesses are replaced by uniform and noncorrelated random numbers between zero and one.

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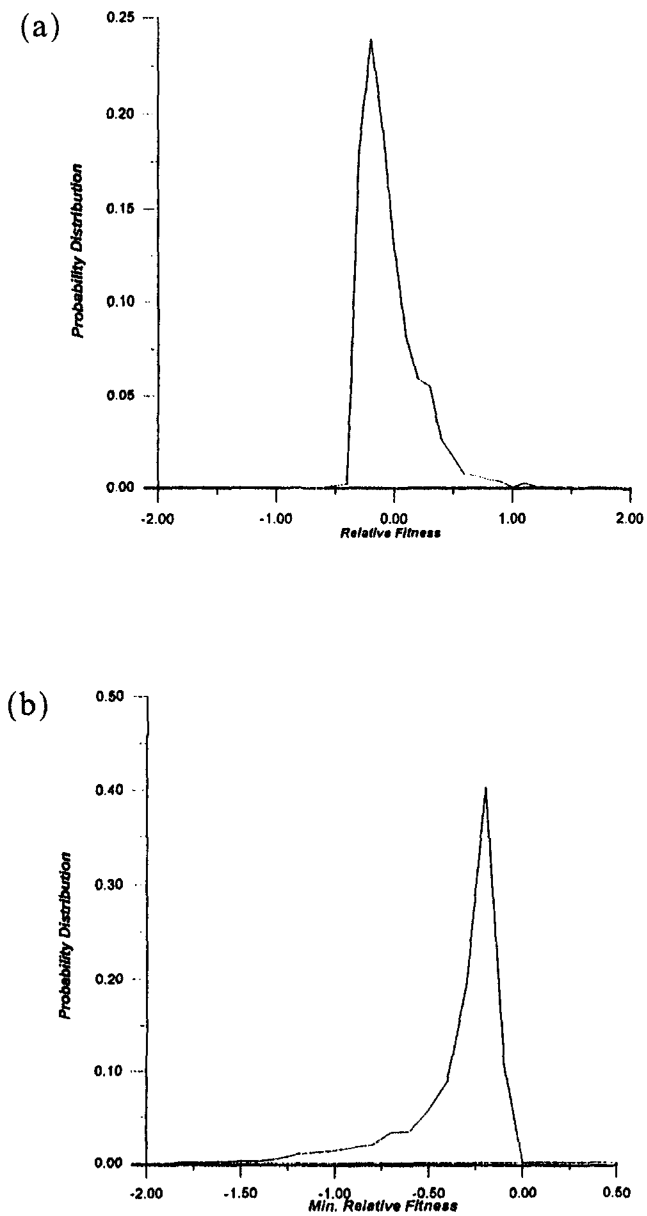



Fig. 1. (a) Relative fitness vs. probability distribution (in $1 \times N$ dimensions); (b) minimum relative fitness vs. probability distribution (in $1 \times N$ dimensions).

Chau *et al.*[6] introduced an alternative model of biological evolution. It deals with the relative fitness of the species, which can be written as

$$v_i = \beta_i(f_i - f_e) + \sum_{k \neq i} \alpha_i(f_i - f_k) \tag{1}$$

for some $\beta_k, \alpha_k \geqslant 0$, where $f_e$ is the fitness of the environment and $f_i$ the fitness of the species $i$. The first term represents the competition of the species $i$ with the environment and the second term represents the competition with other species. For the special case $v_i = 2f_i - f_{i+1} - f_{i-1}$ the boundary conditions are $v_1 = f_1 - f_2$ and $v_n = f_n - f_{n-1}$. The species with the lowest relative fitness $v_i$ together with its two neighbors $v_{i-1}$ and $v_{i+1}$ mutate according to the equations

$$\begin{aligned} v_{i-1} &= v_{i-1} - b_{x,\sigma} \\ v_i &= v_i + 2b_{x,\sigma} \\ v_{i+1} &= v_{i+1} - b_{x,\sigma} \end{aligned} \tag{2}$$

where $b_{x,\sigma}$ is a random number chosen from a Gaussian distribution with mean $x$ and standard deviation $\sigma$. Notice that the random variable which is added and subtracted in the above equations is correlated. This is the difference from the BS model.[5] The updating process is repeated.

In Fig. 1 we restudy their results using uniformly distributed random numbers. The sharp increase in the relative fitness in Fig. 1a is quite similar to the BS model.[5] The similarity between our result and those of ref. 6 confirms that our procedure is adequate despite the limitations we faced in computer facilities.

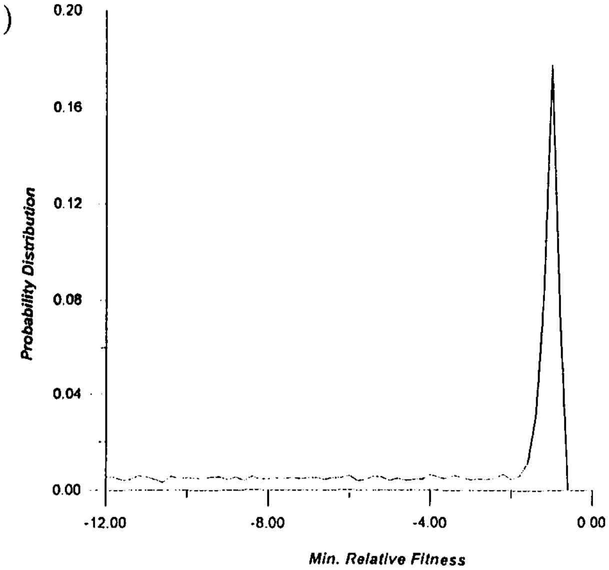
## 2. THE MODEL

In this model only negative interactions e.g., competition, predation, etc., have been included. Since it is known[7] that biological interactions can be negative or positive, e.g., mutualism, sharing, etc., it is important to include both types of interactions. In our model we consider that the species $(i \pm 1, j)$ compete with or predate on the species $(i, j)$, and therefore their fitness will decrease as the fitness of species $(i, j)$ increases. On the other hand, the species $(i, j \pm 1)$ cooperate with the species $(i, j)$, and hence their fitness increases if the fitness of species $(i, j)$ increases.

Our ecosystem consists of $N \times N$ species; the fitness of the species $(i, j)$ is $f_{i,j}$ (a scalar number between zero and one). The relative fitness is given by

$$v_{i,j} = f_{i,j} - f_{i-1,j} - f_{i+1,j} + (f_{i,j-1} + f_{i,j+1})/2 \tag{3}$$

(a)




(b)

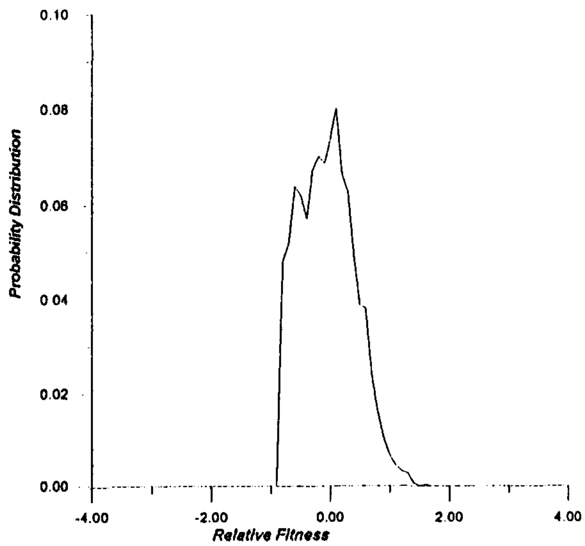



Fig. 2. (a) Relative fitness vs. probability distribution (in $N \times N$ dimensions); (b) minimum relative fitness vs. probability distribution (in $N \times N$ dimensions).

with periodic boundary conditions

$$v_{i,0} = v_{i,n};\ v_{i,n+1} = v_{i,1}$$
$$v_{0,j} = v_{n,j};\ v_{n+1,j} = v_{1,j} \tag{4}$$

If $v_{L,k}$ is the minimum relative fitness in the $N \times N$ ecosystem (the numerics is carried out with $N \times N = 72 \times 72$ species; after $10^4$ iterations the shapes in Fig. 2 are found), it and those of the four nearest neighbors will change by the correlated random number $A$:

$$\begin{aligned} v_{L,k} &= v_{L,k} + A \\ v_{L-1,k} &= v_{L-1,k} - A \\ v_{L+1,k} &= v_{L+1,k} - A \\ v_{L,k-1} &= v_{L,k-1} + A/2 \\ v_{L,k+1} &= v_{L,k+1} + A/2 \end{aligned} \tag{5}$$

Comparison of Figs. 1 and 2 shows that the inclusion of both cooperative interactions, e.g., mutualism, sharing, etc., and antagonistic ones, e.g., competition, predation, etc., increases the fluctuations of the relative fitness, but the singularity behavior is preserved, i.e., all species with $f < f_c$, $f_c \cong -0.9$, die. Notice that $f_c$ is model dependent.

It is interesting to observe the robustness of the singularity behavior and its widespread validity.

## ACKNOWLEDGMENT

One of us (E.A.) wishes to thank Prof. P. Bak for encouragement.

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*Communicated by D. Stauffer*